

# Interschool Test

2019 States

September 2, 2018

## Abstract

This test will hopefully have a little for everyone. Remember to work together, **apply** what you know, and have fun!

## 1 Math

Each of the following questions are worth 10 points. There are sections for a few of the prominent math competition subject areas.

### 1.1 Algebra

1. Saaketh and Swapnil are competing in the final round of the annual HMMT S&S Bee, a battle for the ages. The series they must evaluate is given to them as:

$$\sum_{n=0}^{\infty} \frac{2n+1}{n^2+4n+4}.$$

Swapnil frantically circles his go-to answer of 0. Meanwhile, Saaketh writes down the correct answer, propelling him to international glory. What did Saaketh get as his answer?

2. Find  $y$  if  $y = \sqrt{111 - \sqrt{111 + \sqrt{111 - \dots}}}$ .
3. The zeros of the polynomial  $P(x) = x^3 - 10x + 11$  are  $u$ ,  $v$ , and  $w$ . Determine the value of  $\arctan u + \arctan v + \arctan w$ .
4. Find the smallest positive integer  $n$  for which the polynomial

$$x^n - x^{n-1} - x^{n-2} - \dots - x - 1$$

has a real root greater than 1.999.

### 1.2 Geometry

1. Kishan and Sriraj are playing basketball on a coordinate grid. Kishan is located at  $(3, 0)$ , and the hoop is located at  $(-1, 0)$ . Before Kishan shoots the ball, Sriraj tells Kishan that if he makes a basket, then that basket counts for 5 points instead of 2. Because of this, the hoop moves by the matrix

$$\begin{pmatrix} \frac{1}{2}\sqrt{2+\sqrt{2}} & -\frac{1}{2}\sqrt{2-\sqrt{2}} \\ \frac{1}{2}\sqrt{2-\sqrt{2}} & \frac{1}{2}\sqrt{2+\sqrt{2}} \end{pmatrix}^{2019}$$

to a new position. What is the new distance that Kishan must shoot the ball to make the 5 pointer?

2. Regular dodecagon  $ABCDEFGHIJKL$  has side length 2. Call the midpoint of  $E$  and  $J$  point  $Q$ . Rotating  $Q$  by an angle of  $90^\circ$  around the center of the dodecagon four times, and marking and connecting in clockwise order the four points where  $Q$  lands will result in a figure. Find the area of the figure.

- Sophia, Gabriel, and Vlad think that if they run fast enough in a circle they can form a tornado. They gather in the lobby of the Doubletree and form an equilateral triangle with side length 10 feet. Each person runs toward the person on their right (as each person's position is changing) at a rate of 3 feet per second. What is the area of the figure they form after 1 second?
- The points  $A, B, C, D, E$  lie on a line  $\ell$  in this order. Suppose  $T$  is a point not on  $\ell$  such that  $\angle BTC = \angle DTE$ ,  $\overline{AT}$  is tangent to the circumcircle of  $\triangle BTE$ . If  $AB = 2$ ,  $CD = 15$ , and  $BC = 36$ , compute  $DE$ .

### 1.3 Counting & Probability

- Daniel is creating a crossword puzzle for the meta 2019 States Interschool Test. He refers to the phone book for a few random names to use. The first three names he finds are Akanksha, Aryaan, and Eunice. How many distinct crossword puzzles can he make using these names? (Note that names may not run both adjacent and parallel to one another, and rotations are not distinct.)
- How many ordered quintuples satisfy the equation

$$a_1 + a_2 + a_3 + a_4 + a_5 = 797$$

for  $a_i \geq 10$  and of the form  $2^{k_i} + 1$ , where  $k_i \in \mathbb{Z}$ .

- Jae and Iris are located at the point  $(199, 340, 101)$ . To their dismay, a hurricane blows through and separates them! Jae remains located at  $(199, 340, 101)$ , but Iris is now at the origin. Iris wants to find Jae using the coordinate axes, but the hurricane had also pushed the coordinate axes so they are now transformed by the matrix

$$\begin{pmatrix} 9 & 15 & 8 \\ 7 & 25 & 17 \\ 8 & 5 & 4 \end{pmatrix}.$$

Assuming Iris can only move along the new coordinate "grid" set up, moving forward along one of the three basis vectors at a time (going backward along the basis vectors is not permitted), how many paths are there from Iris to Jae?

- During the 2019 State Convention, Connor, like many others, can be found either at his hotel room, City Walk, his testing room, or Burger King. In moving to and from these four locations, Connor cannot travel directly between Burger King and his hotel room, nor can he travel directly between his testing room and City Walk.

At any given decision between walking to Burger King or his hotel room, he is twice as likely to choose Burger King over his hotel room. However, he is equally likely to walk to City Walk as he is to walk to his testing room on a given decision between the two. Suppose Connor wakes up in Burger King (whoops!) on a busy Saturday morning, and he will change his location 10 times. What is the probability he ends the day in his hotel room?

### 1.4 Calculus

- Find

$$\int (x^6 + x^3) \sqrt[3]{x^3 + 2} dx.$$

- Let  $f(x) = \ln(x+1) + \ln((x+\frac{1}{2})^2 - \frac{5}{4})$  and  $g(x) = \sqrt[3]{e^x - 1}$ . After using Simpson's Rule for a while, a determined Andrew finds that the area,  $R$ , bounded by the graphs of  $f$  and  $g$  is approximately 2.1. Assuming Andrew's approximation is exact, find the volume generated by revolving  $R$  about the line  $y = x + 7$ .
- Compute the integral

$$I = \int_0^1 \sqrt[3]{2x^3 - 3x^2 - x + 1} dx.$$

4. Compute

$$\lim_{n \rightarrow \infty} \left( \frac{2^{\frac{1}{n}}}{n+1} + \frac{2^{\frac{2}{n}}}{n+\frac{1}{2}} + \cdots + \frac{2^{\frac{n}{n}}}{n+\frac{1}{n}} \right).$$

5. Let  $f : [0,1] \rightarrow \mathbb{R}$  be a continuous function. Find the maximum value of the following integral:

$$\int_0^1 x^{2019} \sqrt{xf(x)} dx,$$

given that

$$\int_0^1 f(x) dx = \frac{1}{2}.$$

## 2 Trivia

For this section, solve the questions and match the answers with the characters in the boxes at the bottom of the page. If a character has two numbers listed, then both of these numbers correspond to the letter. Write the letter corresponding to each question on the blanks in the answer sheet. Once all the questions are solved, the letters (read top to bottom) reveal a super secret message. Every correct letter is 2 points, every correct word is 15 points, and the correct message is 50 points (exclude spaces in the answer)!

1. Total number of MAO and FAMAT student delegates in 2018.
2. Number of States that share at least one letter with the word 'doubletree.'
3. Number of characters in  $\text{\LaTeX}$  needed to display euler's identity.
4. Year in which the Adams-Onis treaty was signed.
5. Minimum number of lines needed to depict the meme "loss."
6. Number of weeks that Kanye West's *Yeezus* was on the Billboard 200.
7. Finish the Progression: 15, 30, \_\_, love.
8. Day of year in which Charlemagne established the Holy Roman Emperor.
9. Number of unique scholarships that Mu Alpha Theta offered in 2018.
10. Number of career minutes that NBA Hall of Famer Elvin Hayes played.
11. Number of times the minute hand and hour hand form a 90 degree angle in 24 hours.
12. Number of the question that the smallest percentage of pre-calculus students got correct in 2018.
13. Number of members in the rap group "Migos."
14. Number of solutions to a quadratic when discriminant is negative.
15. Number of prime digits that appear in the first 2019 digits of  $\pi$ .
16. [Average sweepstakes t-score of all schools at state convention in past 7 years.]
17. The current Hebrew year according to the Talmud.
18. Difference in altitude between highest and lowest points on earth in feet.
19. Number of times an NFL team scored using multiple lateral passes during a play.
20. Number of keys on a TI-Nspire CX (the trackpad counts as one key).

A: 17	B: 1	C: 55,000	D: 54,120	E: 12,40	F: 5779 6	G: 80,228	H: 43,46
I: 1819,803	J: 98	K: 318,9	L: 0	M: -3	N: 7,2	O: 18,3	P: 14,6
Q: 599,18001	R: 359	S: 12,2715	T: 8,2	U: 65023	V: 3.75	W: 65021	X: 24
Y: 30,57	Z: 15	?: 1845	!: 71	,: 427	': 10	.: 712	:: 2019

### 3 Orlando

Each of the following trivia questions pertain to Orlando in some sense and is worth 8 points, with an extra bonus of 5 points.

1. Name the tribe that inhabited current-day Orlando before European conquest.
2. Identify the crop that Orlando is best known for producing.
3. Often times, addresses and city borders are not precise. Determine whether the address "7335 W Sand Lake Rd, Orlando, FL" is truly within Orlando city limits. If the address is within Orlando, write "Humuhumunukunukuapua'a" as your answer. Otherwise, tell me the street name that would take me to Orlando in the shortest distance from my address.
4. In close proximity to Orlando is a school very well known for its basketball team. Name this High School that has sent players to elite college basketball programs such as Duke, UCLA, Ohio State, and LSU. (For bonus points [5] name these players referenced in respective order)
5. Ben Wallace and Chauncey Billups are two former elite NBA players who spent parts of their careers with the Orlando Magic. Name a common trait between these two players that was a result of "malice." (hint: what's the opposite of playing basketball?)
6. What is the name given to tornadoes, the smallest of which are about the height of a man, that can found in the central Florida?
7. Name the first theme park built in Orlando.
8. Tiger Woods has won the Arnold Palmer Invitational golf tournament, contested in Orlando, a record 8 times. What two colors does Tiger wear every Sunday of a golf tournament?

# Interschool Test Answer Sheet

School: \_\_\_\_\_ ID: \_\_\_\_\_

## Math

### Algebra

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

### Geometry

1. \_\_\_\_\_

2. \_\_\_\_\_

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4. \_\_\_\_\_

### Counting & Probability

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4. \_\_\_\_\_

### Calculus

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## Trivia

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## Orlando

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